

Final Exam, MTH 512, Spring 2015

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QUESTION 1. (i) Let $T : V \to W$ be a linear transformation such that $dim(V) = dim(W) < \infty$. If T is onto (surjective), prove that T is one to one.

(ii) Given M is a 3×3 matrix and $M \neq I_3$ such that $M^2 = M$. Find all possible rational forms of M and all possible Jordan forms of M.

(iii) Let $S = \{A \in R^{3\times3} \mid A^T = A\}$ and $N = \{B \in R^{3\times3} \mid B^T = -B\}$. Then S, N are subspaces of $R^{3\times3}$ (don't show that). Find dim(S + N). Show the work. Now let $T : R^{3\times3} \to R^{3\times3}$ be a linear transformation such that $T(D) = D$ for every $D \in S$ and $T(F) = F$ for every $F \in N$. Let $H \in R^{3\times3}$. What is T(H)? show the work.

(iv) Let W, H be finite dimensional subspaces of a vector space V such that $W^\perp = H^\perp$. Prove that $W = H$.

(v) Let $A = \begin{bmatrix} cos(t) & sin(t) \\ -sin(t) & cos(t) \end{bmatrix}$ and assume that A is diagnolizable over R for some real number t. Prove that A itself must be a diagonal matrix.

(vi) Let $T = A \oplus B = \begin{bmatrix} 0 & -4 & 0 & 0 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ Find the Jordan Form of T. Find the rational form of T. Find the minimum polynomial of T.

(vii) Given the points $(1, 3), (2, 4), (4, 5)$. Find the best fitting line between them (use the least square method). In order to do that. Consider the normal dot product on R^2.

a. Rewrite the question in the form of $AX = B$.

b. Let S be the column space of A. Find the projection of B over S.

c. Now use (part b) to find the solution or choose another method to find the solution (but all must do part b).

(viii) Give me an example of a 6×6 matrix over R , say A, such that the minimal polynomial of A is $m_A(x) = (x^2 + 1)(x^2 + 4)$. Let B be an $n \times n$ matrix over R where $n \geq 5$ is an odd integer . Prove that it is impossible that $m_B(x) = m_A(x)$.

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